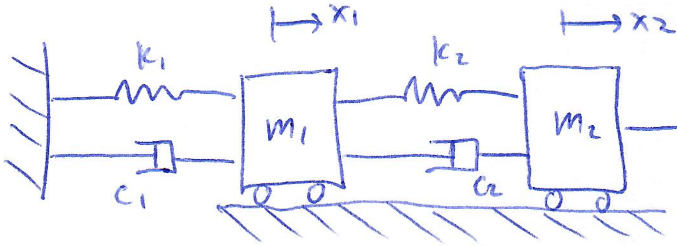
ME 4555 – Lecture 10 – Simscape + linearity

Consider two-mass spring-mass-damper system:



with $m_1 = m_2 = 1$

$c_1 = c_2 = 1$

$k_1 = k_2 = 5$.

Simulink diagram is rather complicated
(see previous lecture for diagram)

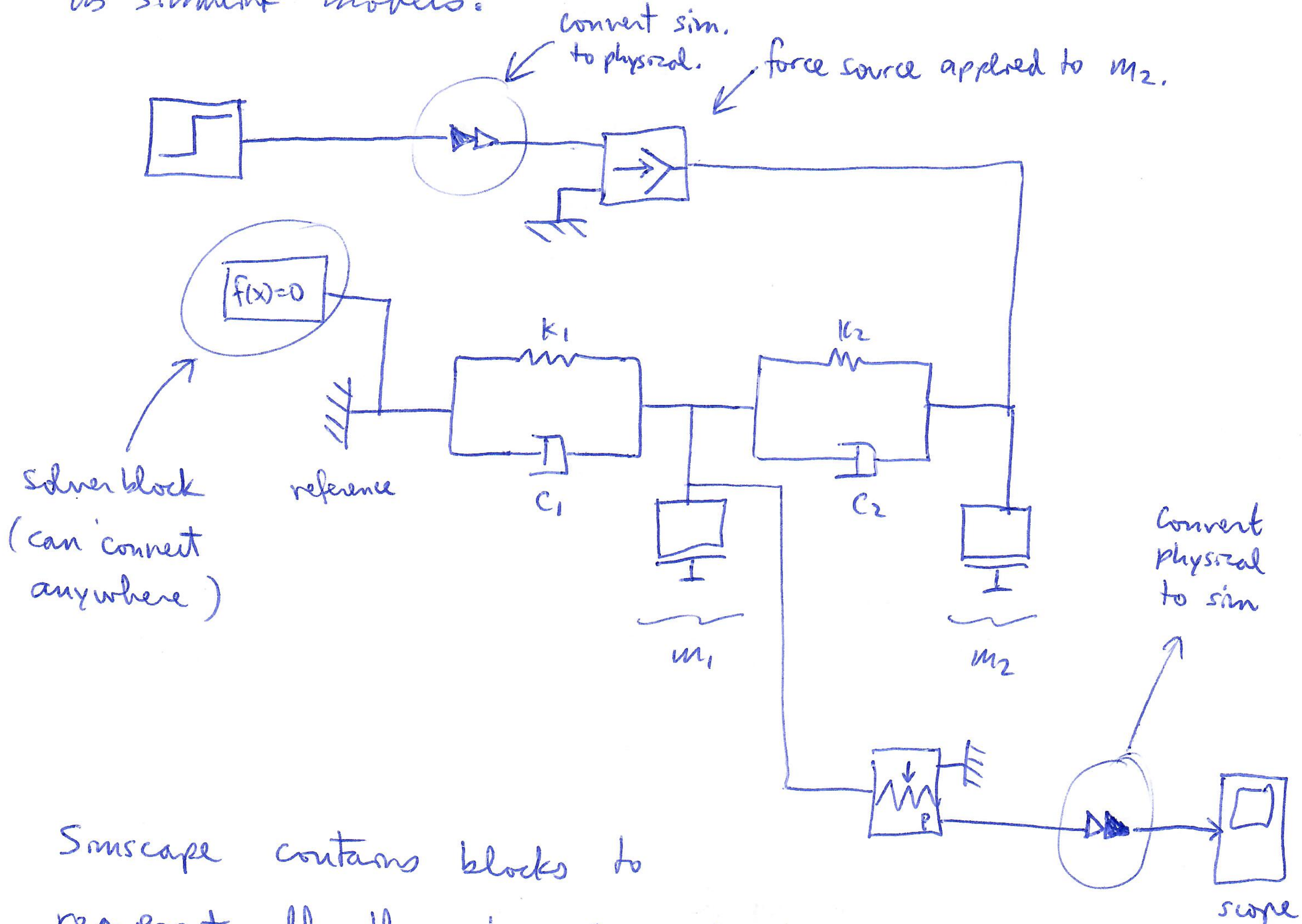
<u>Investigate</u>: (see if you can predict response!)

- ☆ what happens if $k_1 = 100$?
- ☆ what if we also make $c_2 = 2$?
- ☆ what if $m_1 = 10$ and $c_2 = 0.1$?

Complicated systems can be made into <u>subsystems</u> to simplify Simulink model.

New (as of ~2017) functionality in Simulink: Simscape

Allows for a more natural representation of physical systems as simulink models.



Simscape contains blocks to represent all the physical systems we've modeled in class so far:

electrical, mechanical, motors, hydraulic, and more!

The systems we've been studying recently (spring-mass-damper, RLC, ideal motor, op amp, hydraulic) are "linear time-invariant" (LTI) systems.

They have the following 4 properties (assuming zero initial conditions on all states)

1) homogeneity: scaling the input scales the output.

if $u(t) \rightarrow \boxed{G} \rightarrow y(t)$

then $k \cdot u(t) \rightarrow \boxed{G} \rightarrow k \cdot y(t)$

2) superposition: adding inputs adds outputs.

if $u_1(t) \rightarrow \boxed{G} \rightarrow y_1(t)$

and $u_2(t) \rightarrow \boxed{G} \rightarrow y_2(t)$

then $u_1(t) + u_2(t) \rightarrow \boxed{G} \rightarrow y_1(t) + y_2(t)$

these together are called "linearity"

3) time-invariance: delaying input delays output

if $u(t) \rightarrow \boxed{G} \rightarrow y(t)$

then $u(t-\tau) \rightarrow \boxed{G} \rightarrow y(t-\tau)$ for all τ

4) causality: future can't affect the past.

LTI systems have very nice properties we will make use of later on... But how can we check whether a set of ODEs is LTI?

★ it must **only** contain terms of the form

$$\underbrace{a}_{\text{constant}} \cdot \underbrace{x^{(k)}(t)}_{k^{th} \text{ derivative of } x(t)}$$ and they must be added together

$x(t)$ ← or any other signal.

For example:

$m\ddot{x} + c\dot{x} + kx = f$ is linear (LTI)

$m l^2 \ddot{\theta} + mgl \sin\theta = 0$ is not linear (not LTI)

$\sin\theta$ ← nonlinear term.

$m l^2 \ddot{\theta} + mgl\,\theta = 0$ is LTI once we made the small-angle approximation.

Terms like $\dot{\theta}^2$, $\sin\theta$, $\theta\dot{\theta}$, etc... all are nonlinear.

Linearity also works if there are multiple equations, and multiple signals. For example:

motor equation: $\begin{cases} L\dfrac{di}{dt} + Ri + K_b\omega = V_{in} \\ J\dot{\omega} + b\omega - K_t i = -T_L. \end{cases}$

→ all terms are linear, so this is an LTI system.

$$\begin{bmatrix} V_{in} \\ T_L \end{bmatrix} \longrightarrow \boxed{\begin{matrix}\text{motor} \\ \text{(LTI)}\end{matrix}} \longrightarrow \begin{bmatrix} i \\ \omega \end{bmatrix}$$

So, for example, if $\begin{bmatrix} V_{in}^{[1]} \\ T_L^{[1]} \end{bmatrix} \longmapsto \begin{bmatrix} i_1 \\ \omega_1 \end{bmatrix}$

and $\begin{bmatrix} V_{in}^{[2]} \\ T_L^{[2]} \end{bmatrix} \longmapsto \begin{bmatrix} i_2 \\ \omega_2 \end{bmatrix}$ then $\begin{bmatrix} V_{in}^{[1]} + V_{in}^{[2]} \\ T_L^{[1]} + T_L^{[2]} \end{bmatrix} \longmapsto \begin{bmatrix} i_1 + i_2 \\ \omega_1 + \omega_2 \end{bmatrix}$

Can verify directly by combining equations.

$$\left.\begin{aligned} &\text{if } m\ddot{x}_1 + c\dot{x}_1 + kx_1 = f_1 \\ &\text{and } m\ddot{x}_2 + c\dot{x}_2 + kx_2 = f_2 \end{aligned}\right\} \Longrightarrow m\underbrace{(\ddot{x}_1 + \ddot{x}_2)}_{\frac{d^2}{dt^2}(x_1+x_2)} + c\underbrace{(\dot{x}_1 + \dot{x}_2)}_{\frac{d}{dt}(x_1+x_2)} + k(x_1+x_2) = f_1 + f_2$$

so if $f_1(t) + f_2(t)$ is used as input, the solution will be $x_1(t) + x_2(t)$.